



K25U 0158

Reg. No. :

Name :

**Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/
Supplementary/Improvement) Examination, April 2025**

**(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II**

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

(4×1=4)

1. Give an example of a Lipschitz Function.
2. Define Riemann integral of a function $f : [a, b] \rightarrow \mathbb{R}$.
3. State Additivity Theorem.
4. Define Beta function.
5. Find $\lim_{n \rightarrow \infty} \frac{x}{x+n}$ for all $x \in [0, 1]$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

(8×2=16)

6. Define step function on $[a, b]$. Give an example of a step function on $[0, 2]$.
7. Check whether $f(x) = \sin x$ is Lipschitz on $\mathbb{R}$. Justify your answer.
8. Let $f(x) = x^3$ for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition $P = (0, 1, 2, 4)$, take tags at the left end point of the subintervals.
9. Show that every constant function on $[a, b]$ is in $\mathcal{R}[a, b]$.

P.T.O.



10. Let $f \in R[a, b]$, and if $[c, d] \subseteq [a, b]$, show that the restriction of f to $[c, d]$ is in $R[c, d]$.
11. Show that $\int_0^1 \frac{1}{1-x} dx$ diverges.
12. Show that $\Gamma n = (n-1) \Gamma(n-1)$.
13. Compute $\Gamma(-3/2)$.
14. Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f_n(x) = \frac{\sin nx}{n}$. Find the pointwise limit of the sequence of functions (f_n) .
15. Define a metric d on a set S .
16. State Dini's theorem.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

(4×4=16)

17. Define uniformly continuous function. Show that $\sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $(0, 1)$.
18. Show that every continuous function on $[a, b]$ is Riemann integrable.
19. Let $f, g \in R[a, b]$, if $f(x) \leq g(x)$ for all $x \in [a, b]$ then show that $\int_a^b f \leq \int_a^b g$.
20. Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.
21. Show that $\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.
22. Evaluate $\int_0^{\pi/2} (\sin x)^{8/3} (\sec x)^{1/2} dx$.
23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Show that f is continuous on A .



PART – D

Answer **any two** questions. **Each** question carries **six** marks.

(2×6=12)

24. a) Show that a function f is uniformly continuous on the interval (a, b) if and only if it can be defined at the endpoints a and b such that the extended function is continuous on $[a, b]$.

b) Show that $f(x) = \frac{e^x - 1}{x}$ is uniformly continuous on $(0, 1)$.

25. State and prove fundamental theorem of calculus (Second form).

26. a) Show that $\int_1^\infty \frac{1}{x^p} dx = \begin{cases} \frac{1}{p-1}, & p > 1 \\ \infty, & p \leq 1 \end{cases}$.

b) Investigate the convergence of $\int_1^\infty \frac{1 - e^{-x}}{x} dx$.

27. Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f . Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$.

